

Title

On the degenerate Whittaker space for $\mathrm{GL}_4(\mathfrak{o}_2)$.

Abstract

Let $\mathfrak{o}_2$ be a finite principal ideal local ring of length two with residue field $\mathbb{F}_q$. Let $N = \left\{ \begin{pmatrix} I & X \\ 0 & I \end{pmatrix} : X \in M_n(\mathfrak{o}_2) \right\}$. Since $N \cong M_n(\mathfrak{o}_2)$, define a character ψ of N by $\psi(X) := \psi_0(\mathrm{tr}(X))$, where ψ_0 is a primitive character of $\mathfrak{o}_2$. For any representation π of $\mathrm{GL}_{2n}(\mathfrak{o}_2)$, the degenerate Whittaker space $\pi_{N,\psi}$ of π is defined to be the maximal subspace of π on which N acts by ψ . Then $\pi_{N,\psi}$ is a representation of $\mathrm{GL}_n(\mathfrak{o}_2)$.

Our work provides an explicit description of $\pi_{N,\psi}$ for certain representations π of $\mathrm{GL}_4(\mathfrak{o}_2)$. Our work aligns with one of works of Dipendra Prasad on degenerate Whittaker space over finite fields and it extends the understanding of degenerate Whittaker spaces over finite principal ideal local rings.